3
Steady flow in pipes

3.1 Introduction

Pipes are the most frequently used conduits for the conveyance of fluids, both gases and liquids. They are produced in a variety of materials, including steel, cast iron, ductile iron, concrete, asbestos cement, plastics, glass and non-ferrous metals. In their new condition, the finished internal wall surfaces of these glass or plastic surface to the relatively rough concrete surface. Also, depending on the fluid transported and the pipe material, the condition of the pipe wall may vary with time, either due to corrosion, as in steel pipes, or deposition, as in hard water areas.

The fluids of interest in the present context are water, wastewater, sewage sludges and gases such as air, oxygen and biogas. As will be seen later, the flow of water, wastewater and gases in pipes is invariably turbulent. The flow of sludges, however, may well be laminar at practical design velocity values. It is self-evident that fluid density and viscosity are key fluid properties in pipe flow analysis, both obviously having an influence on the power input required to induce flow.

The design engineer is primarily interested in being able to predict accurately the discharge capacity of pipe systems. To do this, it is essential to know the relation between head loss and mean flow velocity.

3.2 Categorisation of pipe flow by Reynolds number

Turbulence in fluid flow is characterised by random local motions, which transfer momentum and dissipate energy. This random motion increases with increase in the mean velocity and is suppressed by solid boundaries. Reynolds (18) carried out extensive pipe flow tests from which he was able to define the flow regime as being either laminar, transitional or turbulent. The flow index which he developed is known as the Reynolds number re , which for pipes, is defined as follows:

$$R_e = \frac{vd\rho}{\mu} \qquad (3.1)$$

where d is the pipe diameter and v is the mean flow velocity. The ranges of values for R_e are as follows:

(1) laminar flow: $R_e \le 2300$;
(2) transitional flow: $2300 \le R_e \le 4000$;
(3) turbulent flow: $R_e \ge 4000$

3.3 Hydraulic and energy grade lines

The flow of real fluids through pipes results in a loss of energy or head along the direction of flow. Referring to Fig. 3.1, the Bernouilli equation can be applied as follows:

$$\frac{p_1}{\rho g} + \frac{\alpha v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha v_2^2}{2g} + z_2 + h_f \qquad (3.2)$$

where h_f is the head loss over the pipe length L. There is always such an energy loss associated with flow. It is graphically represented as a gradient in pressure head i.e. an hydraulic grade line (HGL) or a gradient in energy or "total head", that is, an energy grade line (EGL). In steady uniform flow, as depicted in Fig 3.1, these lines are parallel. The slope of the energy line or friction
slope S is :

$$S_f = \frac{h_f}{L} \tag{3.3}$$

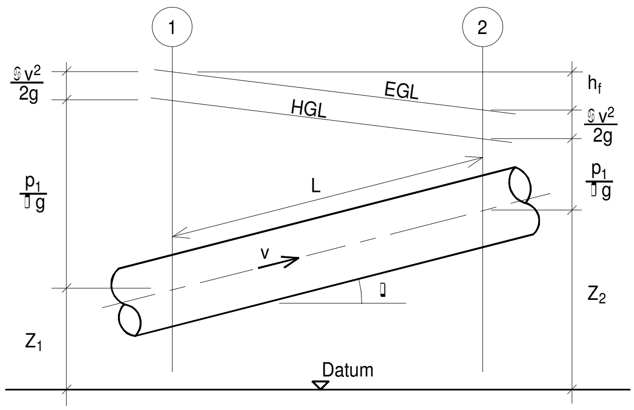


Fig 3.1 **Hydraulic and energy gradients**

3.4 Shear stress distribution

The radial variation of shear stress, under conditions of steady uniform flow, is derived from consideration of the forces acting on the flowing fluid. Since there is no acceleration, the net force acting on the flowing mass of fluid must be zero. The forces acting on the fluid mass between sections 1 and 2 on Fig 3.1 are as follows:

$$p_1 a - \rho g A L \sin\theta - p_2 a - \tau_0 P L = 0 \tag{3.4}$$

where a is the pipe cross-sectional area, τ_0 is the wall shear stress, and P is the section perimeter length. Dividing by ρga and rearranging:

$$\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = \frac{\tau_0 P L}{\rho g a}$$

As may be seen from Fig 3.1, the left-hand side of this equation is equal to the head loss h_f. Therefore

$$h_f = \frac{\tau_0 P L}{\rho g a}$$

Hence

$$\tau_0 = \rho g \frac{a}{P} \frac{h_f}{L}$$

or

$$\tau_0 = \rho g R_h S_f \qquad (3.5$$

where R_h is the hydraulic radius i.e. the ratio of flow area to perimeter length and S_f is the friction slope. Thus, in steady uniform flow, the fluid shear stress at the wall is linearly related to the friction slope, which is a readily measured flow parameter.

The foregoing analysis may also be applied to any concentric cylindrical volume of fluid of smaller diameter than that of the pipe, to give the local fluid shear stress τ_y:

$$\tau_y = \rho g R_y S_f \tag{3.6}$$

where τ_y is the shear stress at a distance y from the pipe wall and R_y is the corresponding hydraulic radius. Thus, the shear stress in pipe flow varies linearly from a maximum value at the pipe wall to zero at its centre.

3.5 Laminar pipe flow

Of primary interest in the hydraulic design of pipe systems is the relationship between carrying capacity and head loss or friction slope. Under laminar flow conditions, the spatial variation in velocity is governed by the fluid viscosity μ and applied shear stress τ:

$$\tau = \mu \frac{dv}{dy}$$

Combining this relationship with eqn (3.6):

$$\rho g \frac{(D-2y)}{4} S_f = \mu \frac{dv}{dy} \tag{3.7}$$

Equation (3.7) can be integrated subject to the boundary conditions that v = 0 at y = 0, to give a parabolic velocity distribution for laminar flow (Fig 3.2):

$$v_y = \frac{\rho g S_f}{4\mu}\left(Dy = y^2\right) \tag{3.8}$$

The maximum velocity at the pipe axis is:

$$v_{max} = \frac{\rho g S_f D^2}{16\mu} \tag{3.9}$$

The mean velocity v is found by integration over the flow cross-section:

$$v = \frac{\int_0^{D/2} v_y \pi (D-2y) dy}{\pi D^2 / 4} \tag{3.10}$$

giving the result:

$$\text{Laminar flow:} \qquad v = \frac{\rho g S_f D^2}{32\mu} \tag{3.11}$$

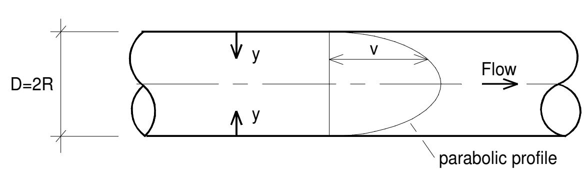


Fig 3.2 **Laminar velocity distribution**

3.6 Turbulent flow in pipes

The random component in turbulent flow renders exact mathematical analysis impossible. However, through a combination of experiment and theoretical reasoning, the magnitude of the resistance to flow of Newtonian fluids under turbulent conditions in pipes has been modelled in mathematical terms, allowing the reliable prediction of head loss for a very wide range of flow and conduit surface conditions. The research works of Nikuradse, Prandtl, von Karman, Colebrook and White, among others, have contributed greatly to this development.

As in laminar flow, the starting point is the velocity distribution over the flow cross-section, which may be expressed in the following form as proposed by Prandtl:

$$\tau = \rho L^2 \left(\frac{dv}{dy}\right)^2 \tag{3.12}$$

where L is the so-called "mixing length", which is not a physical dimension of the system but may be thought of as a measure of the random displacement of fluid elements characteristic of turbulent flow. The value of L has been found to be proportional to y, the distance from the flow boundary:

$$L = Ky \tag{3.13}$$

where K is a numerical constant having a value of approximately 0.4. On insertion of this value for K equation (3.12) may be rewritten in the form:

$$\frac{dv}{dy} = \sqrt{\tau / \rho}\,\frac{2.5}{y} \tag{3.14}$$

It is assumed that the shear stress ç is constant over the flow cross-section under turbulent flow conditions. The term τ/ρ has the dimensions of velocity and is sometimes known as the "shear velocity", denoted by v_* . The velocity distribution over the pipe cross-section is found by integration of equation (3.14):

$$v_y = 2.5\, v_* \ln y + \text{constant} \tag{3.15}$$

This logarithmic velocity distribution clearly cannot be valid at the pipe wall, since ln y has an infinite negative value when y is zero. We may, however, assume that equation (3.15) is valid down to very small values of y, that is, very close to the pipe wall. This condition is satisfied by defining a wall distance y_1, at which the velocity has a zero value. Using this boundary condition, equation (3.15) becomes:

$$v_y = 2.5 v_* \ln\left(\frac{y}{y_1}\right) \tag{3.16}$$

The value of y_1, which may be regarded as effectively defining a new hydraulic boundary inside the actual physical boundary, is determined by flow conditions at the wall.

The turbulent velocity distribution, as represented by equation (3.16), is thus a logarithmic one in which the velocity magnitude varies from a maximum at the centre to zero value at the virtual boundary, as shown on Fig 3.3. The mean velocity is found by integrating the velocity distribution over the cross-section:

$$v = \int_0^{R-y_1} \frac{2\pi\, r v_y\, dr}{\pi R^2}$$

which, on substitution of the right-hand side of equation (3.16) for v_y (noting that y = R-r) and integration, becomes :

$$v = 2.5 v_* \left(\ln\frac{R}{y_1} - 1.5 + 2\frac{y_1}{R} - \frac{y_1^{\,2}}{2R^2}\right)$$

Neglecting terms in y_1/R, which are very small, this expression simplifies to

$$v = 2.5v_*\ln\frac{0.112D}{y_1} \qquad (3.17)$$

Thus, the mean velocity is numerically equal to the local velocity at y = 0.112 D.

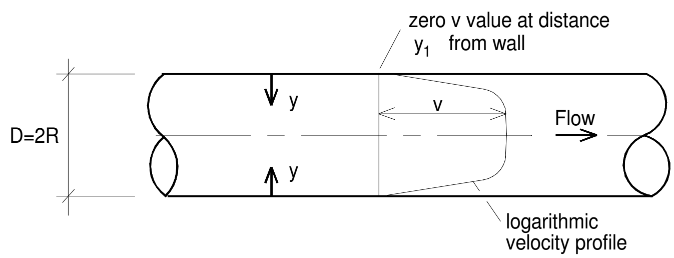


Fig 3.3 **Turbulent velocity distribution**

Where the pipe wall is smooth, as, for example, with glass, plastics and similar surfaces, the flow adjacent to the wall is laminar and fluid drag is exerted on the boundary surface solely by viscous shear. Under such conditions, the magnitude of y is governed by wall shear stress and fluid viscosity and its value has been experimentally found to be:

$$y_1 = \frac{0.1\nu}{v_*} \qquad (3.18)$$

Insertion of this value for y1 in equation (3.17) gives the following value for the mean velocity in **smooth turbulent flow:**

$$\text{Smooth:} \qquad v = 2.5v_*\ln\left(\frac{1.12v_*D}{\nu}\right) \qquad (3.19)$$

The roughness of the internal surfaces of pipes is measured in terms of the "equivalent sand roughness" k (m). This measure of roughness follows from the work of Nikuradse, who used layers of uniform size sand, glued to the internal surface of his experimental pipes, to provide well-defined rough surfaces. Viscosity is found to have a negligible influence on flow when the wall roughness is such that the ratio $k/(\nu/v_*)$ exceeds about 60. Flow under such conditions is described as rough turbulent flow, for which the value of y has been experimentally determined as k/33. Inserting this value for y in equation (3.17) gives the following value for the average velocity in rough **turbulent flow:**

$$\text{Rough:} \qquad v = 2.5v_*\ \ln\left(\frac{3.7D}{k}\right) \qquad (3.20)$$

When the ratio $k/(\nu/v_*)$ is less than 60 and greater than 3, the flow is categorised as being in the transition region between smooth turbulent flow and rough turbulent flow. Flow of water in commercial pipes at conventional velocities is typically in this zone. It is clear that fluid viscosity and wall roughness both influence flow resistance in this transition region between smooth and rough turbulent flow. Colebrook proposed that the effective wall displacement in transition flow be taken as the sum of the wall displacements for smooth and rough flow :

$$y_1 = \frac{0.1\nu}{v_*} + \frac{k}{33} \qquad (3.21)$$

Insertion of this value for y1 in equation (3.17) gives the following expression for the mean velocity in **transition turbulent flow:**

$$\text{Transition:} \qquad v = -2.5 v_* \ln\left(\frac{\nu}{1.12 v_* D} + \frac{k}{3.7D}\right) \tag{3.22}$$

It is clear that the transition expression can be applied over the full regime of turbulent pipe flow. When the pipe roughness is very small, the transition expression approaches the smooth law and, likewise, when the wall displacement associated with the laminar sublayer is small compared with that due to wall roughness, the transition expression approaches the rough law.

3.7 Practical pipe flow computation

3.7.1 The Darcy-Weisbach and Colebrook-White equations

Although the foregoing correlations between mean velocity v and shear velocity v_* may be used directly in pipe flow computation, they are not commonly used in engineering practice. Instead, the findings are incorporated in the Darcy-Weisbach (Darcy 1858, Weisbach 1842) equation, which has the form:

$$S_f = \frac{fv^2}{2gD} \tag{3.23}$$

This empirical formula, which predated the development of the foregoing turbulent pipe flow analysis, has the computational advantage that it incorporates an non-dimensional friction factor or resistance coefficient f. The Darcy-Weisbach equation can be used for all pipe flow categories by treating f as a flow variable, using the previously developed flow resistance equations to determine its value. The advantage of non-dimensionality is retained by expressing viscosity in terms of Reynolds number and pipe roughness k in terms of relative roughness k/D.

The value of f under laminar flow conditions is found by combining equations (3.11) and (3.26):

Laminar flow $\qquad f = \frac{64}{R_e}$ (3.24) $\qquad$ (3.24)

As already noted, the transition equation (3.22) can be used to model flow resistance over the full range of turbulent flow. The corresponding expression for the friction factor f is found by combining equations (3.22) and (3.23) and using the relation:

$$v_* = \sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{gDS_f}{4}}$$

resulting in the following expression which is generally known as the Colebrook-White (1937) equation :

$$\text{Turbulent flow:} \qquad \frac{1}{\sqrt{f}} = -0.88 \ln\left(\frac{k}{3.7D} + \frac{2.5}{R_e\sqrt{f}}\right) \tag{3.25}$$

or

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{k}{3.7D} \frac{2.5}{R_e\sqrt{f}}\right) \tag{3.25a}$$

Thus, the friction factor f is a function of Reynolds number and pipe relative roughness. This functional relationship is illustrated graphically on Fig 3.4, often known as the Moody diagram.

Equations (3.24) and (3.25) together, cover the entire spectrum of pipe flow conditions. Pipe flow computation typically involves the calculation of head loss when the velocity and other relevant parameters are known or the calculation of velocity when the head loss and other relevant parameters are known. Direct computation of f for turbulent flow conditions, using equation (3.25), is not feasible because of the non-explicit form of the equation. An

iterative method of solution must therefore be used. The computer program FRICTF, presented in Section 3.10, uses the interval-halving iterative procedure to calculate f.

Direct computation of velocity is, however, feasible, using the relation

$$f = \frac{2gDS_f}{v^2}$$

Insertion of this value for f in equation (3.25) and (3.25a) gives the following equation, explicit expressions for the velocity v:

ln function: $$v = -0.88\sqrt{2gDS_f}\ \ln\left(\frac{k}{3.7D} + \frac{2.5\nu}{D\sqrt{2gDS_f}}\right) \tag{3.26}$$

log function: $$v = -2.0\sqrt{2gDS_f}\ \log\left(\frac{k}{3.7D} + \frac{2.5\nu}{D\sqrt{2gDS_f}}\right) \tag{3.26a}$$

With the exception of the conveyance of sewage sludge in pipes, which is dealt with in Section 3.8, fluid flow in sanitary engineering is invariably turbulent. The Colebrook-White equation provides the most soundly based correlation of head loss and mean flow velocity in pipes and its adoption is therefore recommended in preference to empirical exponential equations. A number of explicit exponential approximations of the Colebrook-White equation are to be found in the literature (Barr 1975). These may be used where access to a computer is not available.

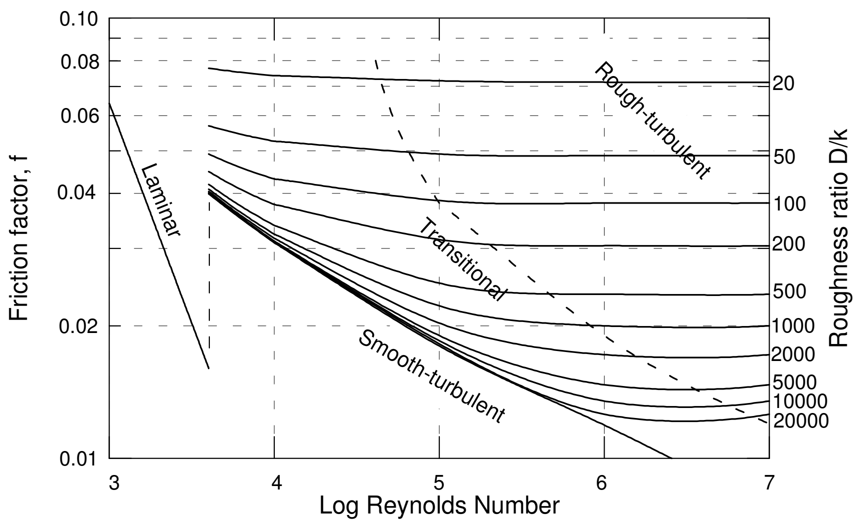


Fig 3.4 Correlation of friction factor with Reynolds number and relative roughness

3.7.2 Design values for pipe roughness

The pipes used for conveying waters and wastewaters are made from a variety of materials with surface finishes varying from the very smooth to the moderately rough. Appropriate design surface roughness or k-value in the "as-new" condition can be read from Table 3.1. The prediction of roughness increase with age more problematic (Colebrook and White 1938; Perkins and Gardiner 1982). Chemically unstable water or wastewater may cause corrosion of metal pipes resulting in surface tuberculation or may give rise to scale deposition which may not only cause increased surface roughness over a period of time but may also significantly reduce the effective pipe bore. Figure 3.5 illustrates both of these phenomena. Waters, such as sewage which contain biodegradable organics, give rise to the growth of a biological slime layer on the inner surfaces of pipes in which they are conveyed. This is a more serious problem in part-filled conduits (discussed in Chapter 7) than in pipes flowing full. The degree of sliming is

inversely related to the flow velocity. Table 3.1 gives recommended equivalent k-values for a range of flow velocities in pipes flowing full.

3.7.3 Other pipe flow equations

The Hazen-Williams formula, published in New York in 1905, is still widely used by water supply engineers. In its original FPS units, it was expressed in the form:

$$v = CR_h^{0.63} S_f^{0.54} \tag{3.27}$$

where C is the Hazen-Williams coefficient. C is a dimensional coefficient and should therefore change its numerical value on conversion to metric units. As this would be confusing for users, the C-value is treated as a pipe constant; conversion to SI units is achieved by introducing a numerical conversion coefficient. The SI form of the Hazen-Williams formula may be written in the form:

$$v = 0.355\, C\, D^{0.63} S_f^{0.54} \tag{3.28}$$

Typical design values for the pipe coefficient C are given in Table 3.2.

The range of applicability of the Hazen-Williams formula may be examined by correlation of the Hazen-Williams C-value and the friction factor f. This correlation is found to be:

$$f = \frac{13.62 g v^{-0.148}}{C^{1.852} D^{0.167}} \tag{3.29}$$

or, expressed in terms of Reynolds number,

$$f = 13.62 g C^{-1.852} R_e^{-0.148} D^{-0.019} \nu^{-0.148} \tag{3.30}$$

which for given values of C, D and ν may be written in the form:

$$\log f = \log C_* - 0.148 \log R_e \tag{3.31}$$

where C_* is a constant. This correlation of f and Reynolds number would plot as a straight line of negative slope on Fig 3.4, showing that the Hazen-Williams formula is valid in the transition turbulent flow zone. It should therefore not be applied in the rough turbulent flow region or when the estimated C-value is less than about 100.

The **Manning** formula (1891,1895) is widely applied to flow in partly filled conduits. It is generally expressed in the form:

$$v = \frac{1}{n} R_h^{0.67} S_f^{0.5} \tag{3.32}$$

expressed in terms of pipe diameter D, this becomes

$$v = \frac{0.397}{n} D^{0.67} S_f^{0.5} \tag{3.33}$$

Typical values for the Manning n roughness coefficient are given in Table 3.3.

Table 3.1
Recommended values for the surface roughness parameter k
(Hydraulics Research, Wallingford, 1990)

Classification (assumed clean and new unless otherwise stated)	Suitable values of k (mm)		
	Good	Normal	Poor
Smooth materials:			
Drawn non-ferrous pipes of aluminium, brass, copper, lead, etc. and non-metallic pipes including plastics, glass etc.	-	0.003	-
Asbestos-cement	0.015	0.03	-
Metal:			
Spun, bitumen-lined, concrete-lined	-	0.03	-
Wrought iron	0.03	0.06	0.15
Uncoated steel	0.015	0.03	0.06
Coated steel	0.03	0.06	0.15
Galvanised iron, coated cast iron	0.06	0.15	0.30
Uncoated cast iron	0.15	0.30	0.60
Tate relined pipes	0.15	0.30	0.60
Old tuberculated water mains with the following degrees of attack:			
Slight	0.60	1.50	3.00
Moderate	1.50	3.00	6.00
Appreciable	6.00	15.00	30.00
Severe	15	30	60
(Good: up to 20 years use; Normal: 40-50 years use; Poor: 80-100 years use)			
Wood:			
Wood stave pipes, planed plank conduits	0.3	0.6	1.5
Concrete:			
Precast concrete pipes with 'O' ring joints	0.06	0.15	0.60
Spun precast concrete pipes with 'O' ring joints	0.06	0.15	0.30
Monolithic construction against steel forms	0.3	0.60	1.50
Monolithic construction against rough forms	0.60	1.50	-
Clayware glazed or unglazed pipe			
With sleeved joints	0.03	0.06	0.15
<150 mm, with spigot and socket joints and 'O' ring seals	-	0.03	-
>150 mm, with spigot and socket joints and 'O' ring seals	-	0.06	-
Pitch fibre	0.003	0.03	-
Glass fibre	-	0.06	-
uPVC			
With chemically cemented joints	-	0.03	-
With spigot and socket joints, 'O' ring seals at 6-9 m centres	-	0.06	-
Brickwork			
Glazed	0.60	1.5o	3.00
Well-pointed	1.5	3.0	6.0
Old, in need of pointing	-	15	30
Sewer rising mains: all materials operating as follows:			
Mean velocity 1 ms^{-1}	0.15	0.3	0.6
Mean velocity 1.5 ms^{-1}	0.06	0.15	0.30
Mean velocity 2 ms^{-1}	0.03	0.06	0.15
Unlined rock tunnels			
Granite and other homogeneous rocks	60	150	300
Diagonally bedded slates	-	300	600

Table 3.2
Hazen-Williams C-values for new pipes

Type of pipe	C-value range
Smooth: plastics, glass, copper, lead, asbestos-cement	135-150
Uncoated cast iron	125-130
Coated metal pipes	135-140
Prestressed concrete (D > 0.5m)	135-145

The region of applicability of the Manning equation may be examined through its correlation with the friction factor f. This correlation is found from equations (3.23) and (3.33):

$$f = 12.7\, g\, n^2\, D^{-0.33} \qquad (3.34)$$

Thus, for given values of D and n, f has a constant value, indicating that the Manning equation is valid for rough turbulent flow and hence should not be used for flow computations relating to smooth pipes.

Table 3.3
Values of Manning n for various types of conduit surface

Surface	Manning n value
Smooth metal	0.010
Smooth concrete	0.012
Rough concrete	0.017
Cut earthen channel	0.025-0.035

3.8 Flow of sewage sludge in pipes

The extent to which the head loss associated with sludge flow in pipes exceeds that for water flow is dependent on both the concentration and nature of the suspended solids. The primary rheological parameter affected by the presence of suspended solids is the fluid viscosity, while the change in density is of lesser significance. Both parameters increase with increase in suspended solids. The flow of sludge in pipes may fall in the laminar, transitional or turbulent flow categories. While laminar flow is not encountered in the practical design flow range used in water distribution, the flow of sludge in pipes is frequently in the laminar range.

The linear correlation of shear stress and shear rate, characteristic of Newtonian fluids, does not apply to sewage sludges at suspended solids concentrations above certain threshold levels. The viscosity of sludge is difficult to measure because of the problem of solids separation and typical thixotropic behaviour i.e. a decrease in viscosity following previous agitation. For many such highly viscous fluids and suspensions, the relation of shear stress and shear rate is non-linear and may be represented as follows:

$$\tau = \tau_y + K\left(\frac{dv}{dy}\right)^n \qquad (3.35)$$

where τ_y is the yield stress, K is a consistency coefficient (Ns^n/m^{-2}) and n is a non-dimensional consistency index. Equation (3.35) is generally known as the Herschel-Bulkley model of fluid flow and is also referred to as the generalised Bingham model of fluid flow. The Herschel-Bulkley model simplifies to the Bingham model when n = 1 and to the so-called power law model when the yield stress $\tau_y = 0$. Recommended guideline values for the various types of sewage sludge, based on data reported by Frost (1983), are given in Table 3.4

3.8.1 Laminar sludge flow in pipes

The flow of sludge in pipes can be classified in laminar/transitional/turbulent categories used a modified Reynolds number criterion, which for a fluid having power law flow characteristics, has the form (Frost 1982):

$$R_e = \frac{\rho v D}{K\{(3n+1)/4n\}^n (8v/D)^{n-1}} \tag{3.36}$$

The term 8v/D is a measure of the wall shear stress (it represents the velocity gradient at the wall in Newtonian fluid flow). The flow is categorised by Reynolds number as follows:

(1)	laminar flow	$R_e < 2300$
(2)	transitional flow:	$2300 < R < 4000$
(3)	turbulent flow:	$R > 4000$

As in all fluid flow, the wall shear stress τ_w is related to the friction slope S_f as follows:

$$\tau_w = \rho g R_h S_f \tag{3.37}$$

where R_h is the hydraulic radius. In laminar flow, the shear stress varies linearly from its maximum value at the pipe wall to zero value at the pipe centre:

$$\tau_r = \tau_w \frac{2r}{D} \tag{3.38}$$

Combining equations (3.35), (3.37) and (3.38) and noting that dv/dy = - dv/dr:

$$-\frac{dv}{dr} = \left[\frac{05\rho g r S_f - \tau_y}{K}\right]^{1/n} \tag{3.39}$$

Clearly, a velocity gradient will only exist where the imposed shear stress $\rho g r S_f/2$ is greater than the yield stress τ_y. Where this is not the case, that is, near the centre of the pipe, there is a region of plug flow. The discharge Q is found by integrating the velocity distribution over the flow cross-section:

$$Q = \int_0^R 2\pi r v_r dr \tag{3.40}$$

where v_r is a function of r, as defined by equation (3.39). Integration of equation (3.40) yields:

$$Q = \frac{\pi D^3}{8}\left(\frac{n}{3n+1}\right)\left(\frac{\tau_w - \tau_y}{K}\right)^{1/n}\left\{1 - \frac{\tau_y/\tau_w}{2n+1}\left[1 + \frac{2n}{n+1}\left(\frac{\tau_y}{\tau_w}\right)\left(1 + \frac{n\tau_y}{\tau_w}\right)\right]\right\} \tag{3.41}$$

The wall shear stress τ_w and hence the friction slope Sf, for a given discharge rate Q, can be found by solution of equation (3.41). This requires knowledge of the flow parameters K, τ_y and n, guideline values for which are given in Table 3.4. The computer program, SSFLO, a listing of which is given at the end of this chapter, uses an interval-halving procedure to solve equation (3.41) for τ_w.

Table 3.4
Guideline values for the rheological parameters K, n and τ_y
C is the sludge solids concentration (kg m^{-3})

Sludge type	K	n	τ_y
Primary	$5.0 \times 10^{-5}C^{2.82}$	$0.79\ C^{-0.17}$	$1.3 \times 10^{-4}C^{2.72}$
Activated	$9.0 \times 10^{-5}C^{3.00}$	$1.70C^{-0.45}$	$1.3 \times 10^{-4}C^{3.00}$
Anaerobically digested	$6.0 \times 10^{-6}C^{3.50}$	$0.90C^{-0.24}$	$1.4 \times 10^{-5}C^{3.37}$
Humus	$2.0 \times 10^{-5}C^{3.00}$	$1.90C^{-0.45}$	$1.6 \times 10^{-5}C^{3.00}$

3.8.2 Turbulent sludge flow in pipes

It has been observed that the head loss in turbulent flow of sludges in pipes can be reliably related to the corresponding head loss for clean water at the same velocity and temperature. The relation is expressed (Frost 1982) in the form of head loss ratio (HLR) factors as follows, where C is the sludge solids concentration (kg m^{-3}):

(1)	Primary sludge:	HLR = 1.5
(2)	Activated sludge	HLR = 0.88 + 0.024 C for C > 5 kg/m^{-3}
(3)	Anaerobically digested sludge	HLR = 0.80 + 0.016 C for C > 15 kg/m^{-3}
(4)	Humus sludge	HLR = 0.80 + 0.020 C for C > 10 kg/m^{-3}

It must be emphasised that the foregoing methods of computation only provide a desk estimate of the head loss due to sludge flow in pipes, which can be used for design purposes in the absence of field measurements the sludge rheological parameters. It is important to note that some sludges may have significantly higher apparent viscosities than determined by the above methods of computation. These include activated sludges which have been mechanically thickened by centrfifuge, dissolved air or flotation process, and also gravity-thickened activated sludge to which polyelectrolyte has been added.

A measurement procedure for the experimental determination of the sludge rheological parameters K, τ_y and n is presented by Frost (1983).

3.9 Head loss in pipe fittings

The total head loss in pipe flow is comprised of the distributed energy loss over straight pipe lengths plus the local losses at bends, tees, valves etc. These local losses may constitute the major part of the total flow resistance in the interconnecting pipework in water and wastewater treatment plants and in the pipework within pumping stations. Poor joint alignment and internal projections associated with welding or gaskets may also contribute significantly to the overall resistance to flow.

The head loss in fittings is usually expressed in terms of the equivalent length of straight pipe or in terms of the velocity head $v^2/2g$. In the latter form, it is expressed as follows:

$$h = K\frac{v^2}{2g} \tag{3.42}$$

where h is the head loss (m), v is the mean pipe velocity (ms^{-1}) and K is a numerical coefficient. The overall head loss for a pipe of length L (m) and diameter D (m) can therefore be expressed as follows:

$$h = \sum K\frac{v^2}{2g} + \frac{fLv^2}{2gD} \tag{3.43}$$

or

$$h = \frac{v^2}{2g}\left(\sum K + \frac{fL}{D}\right) \tag{3.43a}$$

where the summation relates to the K-values for all local losses in the pipe system.

3.9.1 Head losses in valves

A variety of valve types is used in water and wastewater engineering practice. They include gate or sluice valves, butterfly valves, float valves, non-return valves, diaphragm valves, ball valves and pressure-reducing valves. The head loss in flow through these devices depends on the operational position of the device element regulating flow, which may vary from the fully open to the fully closed position. Head loss is also influenced by
The detailed design of the device which may vary from one manufacturer to another. Typical K-values for gate valves, butterfly valves and float valves, over their full operational range (fully open to fully closed) are given in Table 3.5.

3.9.2 Other pipe fittings

In this context, the designation pipefitting includes pipe junctions, bends, pipe entry and exit. Recommended K-values are presented in Table 3.6. It should be noted that where there is a velocity change in flow through a fitting, the velocity to which the K-value is attached is indicated on the sketch of the fitting.

Table 3.5
Typical K-values for valves
(These K-values are for use in eqn (3.42), where v is the computed velocity based on a fully open valve)

% Open	Butterfly*	Gate	Float
100	0.3	0.1	4.2
90	0.5	0.2	4.8
80	0.9	0.4	5.5
70	2.0	0.8	6.6
60	5.0	1.7	8.5
50	14	3.3	12
40	50	5.8	19
30	70	10	41
20	220	23	171
10	2200	80	2500
0	(valve fully closed, zero flow		

*% open relates to angle of rotation of valve disc, for example, 50% open = 45^o disc rotation

3.9.3 Head loss in flow of sludge through fittings

The hydraulic resistance to the flow of sludge through pipe fittings, such as bends, tees etc., can be correlated with pipe velocity in the same manner as outlined for clean water:

$$h = K_s \frac{v^2}{2g}$$

where K_s is the sludge head loss coefficient, which can be related (Frost, 1982, 1983) to the corresponding K-value for clean water (Table3.6):

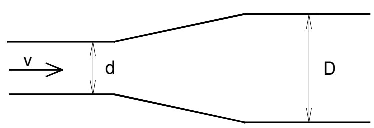
$$K_s = K\left(1 + \frac{2000}{R_e}\right) \tag{3.44}$$

where R_e is the sludge Reynolds number, as defined by equation (3.36). Thus, at low Reynolds numbers, that is, under laminar flow conditions, the hydraulic resistance to flow through pipework fittings considerably exceeds that for clean water.

Table 3.6
Head loss in pipes fittings: K-factors
Head loss (m) = $K(v^2/2g)$, where v is the mean pipeline velocity (ms^{-1})

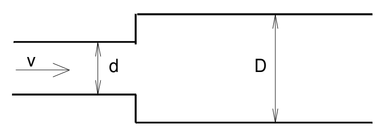
Tapers

Taper: Expanding flow



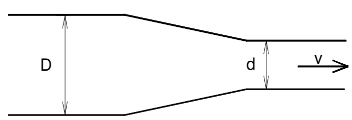
D/D	0.5	0.6	0.7	0.8	0.9
K	0.75	0.50	0.25	0.10	0

Sudden enlargement



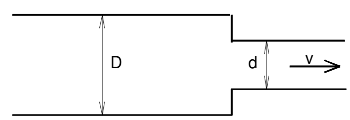
d/D	0.2	0.35	0.5	0.65	0.8
K	1.0	0.3	0.6	0.35	0.15

Taper: Contracting flow



D/D	0.5	0.6	0.7	0.8	0.9
K	0.2	0.17	0.1	0.05	0

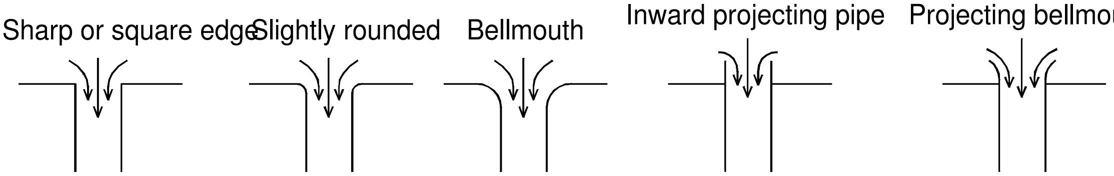
Sudden contraction



d/D	0.5	0.6	0.7	0.8
K	0.5	0.45	0.35	0.2

Entry losses

	Sharp or square edge	Slightly rounded	Bellmouth	Inward projecting pipe	Projecting bellmoutl

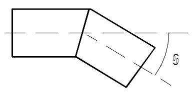


K	0.50	0.25	0.10	0.80	0.20

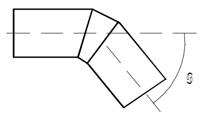
Bends

Type	K
90o short radius	0.40
90o long radius	0.35
45o short radius	0.20
45o long radius	0.17
90o elbow bends	1.25
45o elbow bends	0.50

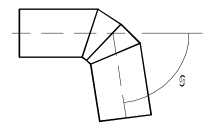
Mitre bends



α	K
90°	1.20
80°	1.00
70°	0.80
60°	0.60
50°	0.40
40°	0.30
30°	0.15
20°	0.10
10°	0.05



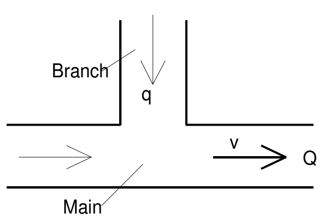
α	K
60°	0.25
45°	0.20
30°	0.15



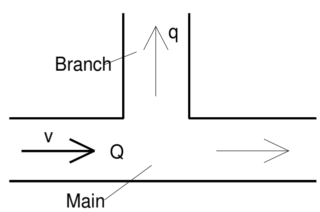
α	K
90°	0.30
75°	0.25
60°	0.20

Square-edged tees

Combining flow



Dividing flow



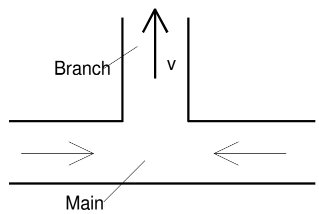
Combining flow:

Flow ratio	Diameter ratio (branch/main)		
q/Q	0.5	0.75	1.0
	Headloss in line		
0	0.1	0.1	0.1
0.25	0.4	0.4	0.4
0.50	0.7	0.6	0.5
0.75	1.0	0.8	0.6
	Headloss branch to main		
0.25	0.7	0	-0.2
0.50	3.5	0.9	0.5
0.75	7.0	2.0	0.9
1.00	11.0	3.0	1.2

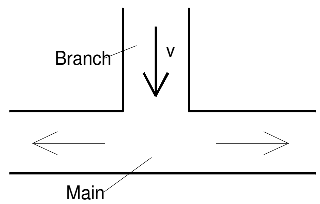
Dividing flow:

Flow ratio	Diameter ratio (branch/main)		
q/Q	0.50	0.75	1.0
	Headloss in line		
0	0.1	0.1	0.1
0.25	0	0	0
0.50	0	0	0
0.75	0.2	0.2	0.2
	Headloss branch to main		
0.25	2.2	1.0	0.9
0.50	6.5	1.3	0.9
0.75	11.0	1.7	1.1
1.00	14.0	2.3	1.3

Combining equal flows
Diameter ratio (branch/main) = 1
K = 0.7

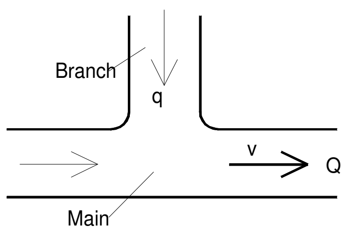


Dividing flow equally
Diameter ratio (branch/main) = 1
K = 1.8

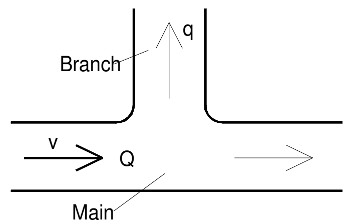


Radiused tees

Combining flow



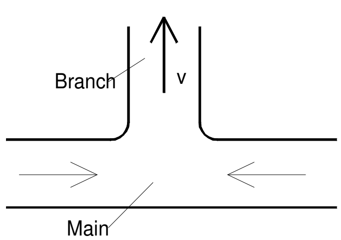
Dividing flow



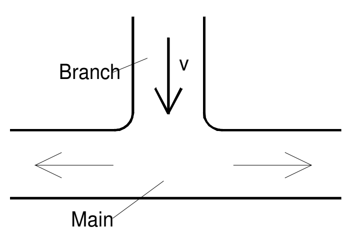
Flow ratio q/Q	Diameter ratio (branch/main) 0.5	0.75	1.0
	Headloss in line		
0	0.1	0.1	0.1
0.25	0.3	0.3	0.3
0.50	0.4	0.3	0.3
0.75	0.2	0.1	0.1
	Headloss branch to main		
0.25	0.7	0	-0.2
0.50	1.4	0.4	0.2
0.75	3.5	0.7	0.4
1.00	8.3	2.0	0.7

Flow ratio q/Q	Diameter ratio (branch/main) 0.50	0.75	1.0
	Headloss in line		
0	0.1	0.1	0.1
0.25	0	0	0
0.50	0	0	0
0.75	0.2	0.2	0.2
	Headloss branch to main		
0.25	1.5	0.8	0.4
0.50	2.8	0.8	0.6
0.75	3.9	0.8	0.6
1.00	4.9	1.0	0.7

Combining equal flows
Diameter ratio (branch/main) = 1
K = 0.4



Dividing flow equally
Diameter ratio (branch/main) = 1
K = 1.2



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